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Foundations and scope of chiral perturbation theory

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Abstract

The aim of this introductory lecture is to review the arguments, according to which the symmetry properties of the strong interaction reveal themselves at low energies. I first discuss the symmetries of QCD, then sketch the method used to work out their implications and finally take up a few specific issues, where new experimental results are of particular interest to test the predictions.

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Chromodynamics is a gauge theory. The form of the interaction among the gluons and quarks is fully determined by gauge invariance. This implies, in particular, that the various different quark flavours, u, d, \dots interact with the gluons in precisely the same manner. As far as the strong interaction is concerned, the only distinction between, say, an s -quark and a c -quark is that the mass is different. In this respect, the situation is the same as in electrodynamics, where the interaction of the charged leptons with the photon is also universal, such that the only difference between e, μ and τ is the mass. As an immediate consequence, the properties of a bound state like the $\Lambda_s = (uds)$ are identical with those of the $\Lambda_c = (udc)$, except for the fact that m_c is larger than m_s .

1 Isospin symmetry

A striking property of the observed pattern of bound states is that they come in nearly degenerate *isospin* multiplets: (p, n) , (π^+, π^0, π^-) , \dots . In fact, the splittings within these multiplets are so small that, for a long time, isospin was taken for an *exact* symmetry of the strong interaction; the observed small mass difference between neutron and proton or K^0 and K^+ was blamed on the electromagnetic interaction. We now know that this picture is incorrect: the bulk of isospin breaking does not originate in the electromagnetic fields, which surround the various particles, but is due to the fact that the d -quark is somewhat heavier than the u -quark.

From a theoretical point of view, the quark masses are free parameters — QCD makes sense for any value of m_u, m_d, \dots . It is perfectly legitimate to compare the real situation with a theoretical one, where some of the quark masses are given values, which differ from those found in nature. In connection with isospin symmetry, the theoretical limiting case of interest is a fictitious world, with $m_u = m_d$. In this limit, the flavours u and d become indistinguishable. The Hamiltonian acquires an exact symmetry with respect to the transformation

$$\begin{aligned} u &\rightarrow \alpha u + \beta d \\ d &\rightarrow \gamma u + \delta d \end{aligned} , \quad V = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} ,$$

provided the 2×2 matrix V is unitary, $V \in U(2)$. Even for $m_u \neq m_d$, the Hamiltonian of QCD is invariant under a change of phase of the quark fields.

The extra symmetry, occurring if the masses of u and d are taken to be the same, is contained in the subgroup $SU(2)$, which results if the phase of the matrix V is subject to the condition $\det V = 1$. The above transformation law states that u and d form an isospin doublet, while the remaining flavours s, c, \dots are singlets.

In reality, m_u differs from m_d . The isospin group $SU(2)$ only represents an *approximate* symmetry. The piece of the QCD Hamiltonian, which breaks isospin symmetry, may be exhibited by rewriting the mass term of the u and d quarks in the form

$$m_u \bar{u}u + m_d \bar{d}d = \frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) + \frac{1}{2}(m_d - m_u)(\bar{d}d - \bar{u}u) .$$

The remainder of the Hamiltonian is invariant under isospin transformations and the same is true of the operator $\bar{u}u + \bar{d}d$. The QCD Hamiltonian thus consists of an isospin invariant part \bar{H}_0 and a symmetry breaking term \bar{H}_{sb} , proportional to the mass difference $m_d - m_u$,

$$H_{\text{QCD}} = \bar{H}_0 + \bar{H}_{\text{sb}} , \quad \bar{H}_{\text{sb}} = \frac{1}{2}(m_d - m_u) \int d^3x (\bar{d}d - \bar{u}u) . \quad (1)$$

The strength of isospin breaking is controlled by the quantity $m_d - m_u$, which plays the role of a *symmetry breaking parameter*. The fact that the multiplets are nearly degenerate implies that the operator \bar{H}_{sb} only represents a small perturbation — the mass difference $m_d - m_u$ must be very small. QCD thus provides a remarkably simple explanation for the fact that the strong interaction is nearly invariant under isospin rotations: it so happens that the difference between m_u and m_d is small — this is all there is to it.

The symmetry breaking also shows up in the properties of the vector currents, e.g. in those of $\bar{u}\gamma^\mu d$. The integral of the corresponding charge density over space, $I^+ = \int d^3x u^\dagger d$, is the isospin raising operator, converting a d -quark into a u -quark. The divergence of the current is given by

$$\partial_\mu(\bar{u}\gamma^\mu d) = i(m_u - m_d)\bar{u}d , \quad (2)$$

and only vanishes for $m_u = m_d$, the condition for the charge I^+ to be conserved. In the symmetry limit, there are three such conserved charges, the three components of isospin, $\vec{I} = (I^1, I^2, I^3)$. The isospin raising operator considered above is the combination $I^+ = I^1 + iI^2$. Since \bar{H}_0 is invariant under isospin rotations, it conserves isospin,

$$[\vec{I}, \bar{H}_0] = 0 . \quad (3)$$

2 Eightfold way

On the basis of the few strange particles, which had been discovered in the course of the 1950's, Gell-Mann and Ne'eman [1] inferred that the strong interaction possesses a further approximate symmetry, of the same qualitative nature as isospin, but more strongly broken. The symmetry, termed the *eightfold way*, played a decisive role in unravelling the quark degrees of freedom. By now, it has become evident that the mesonic and baryonic levels are indeed grouped in multiplets of SU(3) — singlets, octets, decuplets — and there is also good phenomenological support for the corresponding symmetry relations among the various observable quantities.

In the framework of QCD, eightfold way symmetry occurs in the theoretical limit, where the three lightest quarks are given the same mass, $m_u = m_d = m_s$. The Hamiltonian then becomes invariant under the transformation

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \rightarrow V \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad V \in \text{SU}(3)$$

of the quarks fields and the spectrum of the theory consists of degenerate multiplets of this group. The degeneracy is lifted by the mass differences $m_s - m_d$ and $m_d - m_u$, which represent the symmetry breaking parameters in this case. Since the eightfold way does represent an approximate symmetry of the strong interaction, both of these mass differences must be small. Moreover, the observed level pattern requires $|m_d - m_u| \ll |m_s - m_d|$.

Formally, the above discussion may be extended to include additional flavours. One may even consider the theoretical limit, where all of the quarks are given the same mass. The extension, however, does not correspond to an approximate symmetry. The lightest pseudoscalar bound state with the quantum numbers of $\bar{d}c$, e.g., sits at $M_{D^+} \simeq 1.87 \text{ GeV}$. If the mass of the charmed quark is set equal to m_u , this state becomes degenerate with the π^+ . Clearly, the mass difference $m_c - m_u$, which plays the role of a symmetry breaking parameter in this case, does not represent a small perturbation. We do not know why the quark masses follow the pattern observed in nature, nor do we understand the equally queer pattern of lepton masses. It so happens that the mass differences between u , d and s are small, such that the eightfold way represents a decent approximate symmetry.

3 Chiral symmetry

The approximate symmetries discussed above explain why the bound states of QCD exhibit a multiplet pattern, but they do not account for an observation which is equally striking and which plays a crucial role in strong interaction physics — the mass gap of the theory, M_π , is remarkably small. The approximate symmetry, hiding behind this observation, was discovered by Nambu [2]. It originates in a phenomenon, which is well-known from neutrino physics: right- and left-handed components of *massless* fermions do not communicate.

The symmetry, which forbids right-left-transitions, manifests itself in the properties of the axial vector currents, such as $\bar{u}\gamma^\mu\gamma_5 d$. The corresponding continuity equation reads

$$\partial_\mu(\bar{u}\gamma^\mu\gamma_5 d) = i(m_u + m_d)\bar{u}\gamma_5 d . \quad (4)$$

While the divergence of the vector current $\bar{u}\gamma^\mu d$ is proportional to the difference $m_u - m_d$, the one of the axial current is proportional to the sum $m_u + m_d$. If the two masses are set equal, the vector current is conserved and the Hamiltonian becomes symmetric with respect to isospin rotations. If they are not only taken equal, but equal to zero, then the axial current is conserved, too, such that the corresponding charge $I_5^+ = \int d^3x d^\dagger \gamma_5 u$ also commutes with the Hamiltonian — QCD acquires an additional symmetry.

The isospin operator I^+ converts a d -quark into a u -quark, irrespective of the helicity. The operator I_5^+ , however, acts differently on the right- and left-handed components. The sum $\frac{1}{2}(I^+ + I_5^+)$ takes a righthanded d -quark into a righthanded u -quark, but leaves left-handed ones alone. This implies that, for massless quarks, the Hamiltonian is invariant with respect to a set of *chiral* transformations: independent isospin rotations of the right- and left-handed components of u and d ,

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} \rightarrow V_R \begin{pmatrix} u_R \\ d_R \end{pmatrix} , \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix} \rightarrow V_L \begin{pmatrix} u_L \\ d_L \end{pmatrix} , \quad V_R, V_L \in \text{SU}(2) .$$

The corresponding symmetry group is the direct product of two separate isospin groups, $\text{SU}(2)_R \times \text{SU}(2)_L$. The symmetry is generated two sets of isospin operators: ordinary isospin, \vec{I} and chiral isospin, \vec{I}_5 . The particular operator considered above is the linear combination $I_5^+ = I_5^1 + i I_5^2$.

In reality, chiral symmetry is broken, because m_u and m_d do not vanish. As above, the Hamiltonian may be split into a piece which is invariant under the symmetry group of interest and a piece which breaks the symmetry. In the present case, the symmetry breaking part is the full mass term of the u and d quarks,

$$H_{\text{QCD}} = H'_0 + H'_{\text{sb}} , \quad H'_{\text{sb}} = \int d^3x (m_u \bar{u}u + m_d \bar{d}d) . \quad (5)$$

The symmetric part conserves ordinary as well as chiral isospin,

$$[\vec{I}, H'_0] = 0 , \quad [\vec{I}_5, H'_0] = 0 . \quad (6)$$

Note that the symmetry group exclusively acts on u and d — the remaining quarks s, c, \dots are singlets. The corresponding mass terms $m_s \bar{s}s + m_c \bar{c}c + \dots$ do not break the symmetry and are included in H'_0 .

4 Spontaneous symmetry breakdown

Much before QCD was discovered, Nambu pointed out that chiral symmetry breaks down spontaneously. The phenomenon plays a crucial role for the properties of the strong interaction at low energy. To discuss it, I return to the theoretical scenario, where m_u and m_d are set equal to zero.

In this framework, isospin is conserved. The isospin group $SU(2)$ represents the prototype of a "manifest" symmetry, with all the consequences known from quantum mechanics: (i) The energy levels form degenerate multiplets. (ii) The operators \vec{I} generate transitions within the multiplets, taking a neutron, e.g., into a proton, $I^+|n\rangle = |p\rangle$. (iii) The ground state is an isospin singlet,

$$\vec{I}|0\rangle = 0 . \quad (7)$$

If chiral symmetry was realized in the same manner, the energy levels would be grouped into degenerate multiplets of the group $SU(2)_R \times SU(2)_L$. Since the chiral isospin operators \vec{I}_5 carry negative parity, the multiplets would then necessarily contain members of opposite parity. The listings of the Particle Data Group, however, do not show any trace of such a pattern. A particle with the quantum numbers of $I_5^+|n\rangle$ and nearly the same mass as the neutron, e.g., is not observed in nature.

In fact, the symmetry of the Hamiltonian does not ensure that the corresponding eigenstates form multiplets of the symmetry group. In particular, the state with the lowest eigenvalue of the Hamiltonian need not be a singlet. In the case of a magnet, e.g., the Hamiltonian is invariant under rotations of the spin directions, but the ground state fails to be invariant, because the spins are aligned and thereby single out a direction. Whenever the state with the lowest eigenvalue is less symmetric than the Hamiltonian, the symmetry is called "spontaneously broken" or "hidden". Chiral symmetry belongs to this category. For dynamical reasons, the most important state — the vacuum — is symmetric only under ordinary isospin rotations, but does not remain invariant if a chiral rotation is applied,

$$\vec{I}_5 |0\rangle \neq 0 . \quad (8)$$

Since the Hamiltonian commutes with chiral isospin, the three states $\vec{I}_5 |0\rangle$ have the same energy as the vacuum, $E = 0$. The operators \vec{I}_5 do not carry momentum, either, so that the states $\vec{I}_5 |0\rangle$ have $\vec{P} = 0$. This indicates that the spectrum of physical states contains three massless particles. Indeed, the Goldstone theorem [3] rigorously shows that spontaneous symmetry breakdown gives rise to massless particles, "Goldstone bosons". Their quantum numbers are those of the states $\vec{I}_5 |0\rangle$: spin zero, negative parity and $I = 1$.

The three lightest mesons, π^+, π^0, π^- , carry precisely these quantum numbers. The chiral isospin operators act like creation or annihilation operators for pions: Applied to the vacuum, they generate a state containing a pion, $I_5^+ |0\rangle = |\pi^+\rangle$. Applied to a neutron, they do not lead to a parity partner, but instead yield a state containing a neutron and a pion, $I_5^+ |n\rangle = |n\pi^+\rangle$, etc.

5 Pion mass

The above discussion concerns the theoretical world, where u and d are assumed to be massless, such that the group $SU(2)_R \times SU(2)_L$ represents an exact symmetry. The Hamiltonian of QCD contains a quark mass term, which breaks chiral symmetry. To see how this affects the mass of the Goldstone bosons, consider the transition matrix element of the axial current $\bar{u}\gamma^\mu\gamma_5 d$, from the vacuum to a one-pion state. Lorentz invariance implies that this

matrix element is determined by the pion momentum p^μ , up to a constant,

$$\langle \pi^+(p) | \bar{u}(x) \gamma^\mu \gamma_5 d(x) | 0 \rangle = -ip^\mu \sqrt{2} F_\pi e^{ipx} .$$

The value of the constant is measured in pion decay, $F_\pi \simeq 93$ MeV. For the divergence $\partial_\mu (\bar{u} \gamma^\mu \gamma_5 d)$, this yields an expression proportional to $p^2 = M_{\pi^+}^2$. Denoting the analogous matrix element of the pseudoscalar density by G_π ,

$$\langle \pi^+(p) | \bar{u}(x) \gamma_5 d(x) | 0 \rangle = i\sqrt{2} G_\pi e^{ipx} ,$$

the conservation law (4) thus implies the exact relation

$$M_{\pi^+}^2 = (m_u + m_d) (G_\pi / F_\pi) . \quad (9)$$

The relation confirms that, when the symmetry breaking parameters m_u, m_d are put equal to zero, the pion mass vanishes, independently of the masses of the other quark flavours. The group $SU(2)_R \times SU(2)_L$ then represents a spontaneously broken, *exact* symmetry, with three strictly massless Goldstone bosons. When the quark masses are turned on, the Goldstone bosons pick up mass: M_{π^+} grows in proportion to $\sqrt{m_u + m_d}$. The pions remain light, provided m_u and m_d are small. The quark mass term of the Hamiltonian then amounts to a small perturbation, such that the group $SU(2)_R \times SU(2)_L$ still represents an *approximate* symmetry, with approximately massless Goldstone bosons.

Moreover, as noted in section 2, the observed level pattern also requires the differences between m_u, m_d and m_s to be small. Hence the strange quark must be light, too, such that the corresponding mass term may also be treated as a perturbation. The decomposition of the Hamiltonian then takes the form

$$H_{\text{QCD}} = H_0 + H_{\text{sb}} , \quad H_{\text{sb}} = \int d^3x (m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s) . \quad (10)$$

The first term, H_0 , describes three massless flavours (u, d, s) as well as three massive ones (c, b, t). It is symmetric with respect to independent rotations of the right- and left-handed components of u, d and s , i.e., with respect to the group $SU(3)_R \times SU(3)_L$. The perturbation series, which results if H_{sb} is treated as a perturbation, amounts to an expansion of the matrix elements and eigenvalues in powers of m_u, m_d and m_s . The inequality $|m_d - m_u| \ll |m_s - m_d|$, which follows from the fact that isospin breaking is much smaller

than the breaking of eightfold way symmetry, implies that the s -quark is considerably heavier than the other two, $m_u, m_d \ll m_s$.

The above arguments rely on two phenomenological observations:

- (a) The pion mass is small compared to the masses of all other hadrons. This indicates that the strong interaction possess an approximate, spontaneously broken symmetry, with the pions as the corresponding Goldstone bosons. Indeed, the Hamiltonian of QCD exhibits an approximate symmetry with the proper quantum numbers, provided both m_u and m_d are small.
- (b) The multiplet structure seen in the particle data tables indicates that the eightfold way is an approximate symmetry of the strong interaction. For QCD to possess such a symmetry, the mass differences $m_d - m_u$ and $m_s - m_d$ must be small.

Combining the two observations, one concludes that the mass of the strange quark also amounts to a small perturbation: The two groups $SU(3)$ and $SU(2)_R \times SU(2)_L$ can be approximate symmetries of the QCD Hamiltonian only if $SU(3)_R \times SU(3)_L$ represents an approximate symmetry, too. The masses of the other quarks occurring in the Standard Model, on the other hand, cannot be treated as a perturbation. Since the corresponding fields $c(x)$, $b(x)$ and $t(x)$ are singlets with respect to $SU(3)_R \times SU(3)_L$, their contribution may be included in the symmetric part of the Hamiltonian, H_0 . Their presence does not significantly affect the low energy structure of the theory.

The decomposition of the QCD Hamiltonian in eq. (10) may be compared with the standard perturbative splitting

$$H_{\text{QCD}} = H_{\text{free}} + H_{\text{int}} ,$$

where the first term describes free quarks and gluons, while the second accounts for their interaction. The corresponding expansion parameter is the coupling constant g . Since QCD is asymptotically free, the effective coupling becomes weak at large momentum transfers — processes which exclusively involve large momenta may indeed be analyzed by treating the interaction as a perturbation. Perturbation theory, however, fails in the low energy domain, where the effective coupling is strong, such that it is not meaningful to truncate the expansion in powers of H_{int} after the first few terms. In particular, the structure of the ground state cannot be analyzed in this way, while the above decomposition, which retains the interaction among the quarks and

gluons in the "unperturbed" Hamiltonian H_0 and only treats m_u , m_d and m_s as perturbations, is perfectly suitable for that purpose. Note that the character of the perturbation series in powers of H_{sb} is quite different from the one in powers of H_{int} : while the eigenstates of H_{free} are known explicitly, this is not the case with H_0 , which still describes a highly nontrivial, interacting system. H_0 differs from the full Hamiltonian only in one respect: it possesses an exact group of chiral symmetries.

6 Quark masses

There is an immediate experimental check of the above theoretical arguments: the spontaneous breakdown of the symmetry $SU(3)_R \times SU(3)_L$ to the subgroup $SU(3)_{R+L}$ generates eight Goldstone bosons. They are not massless, because the quark masses m_u , m_d and m_s break the symmetry, but since the breaking is supposed to be small, these levels should remain lowest. Indeed, the eight lightest bound states, π^+ , π^0 , π^- , K^+ , K^0 , \bar{K}^0 , K^- , η , do carry the required quantum numbers, both with respect to spin/parity and to flavour.

As a further confirmation of the picture, one may compare the mass splittings within the pseudoscalar octet with those of the other multiplets. The mass *differences* are comparable: $M_\eta - M_\pi \simeq 410$ MeV, $M_\Xi - M_N \simeq 380$ MeV. The mass *ratios* of the Goldstone bosons, however, deviate much more strongly from unity than those of the other multiplets: while the various levels of the baryon octet differ from their mean mass by less than 20 %, the mass of the η is four times as large as the mass of the pion. The above symmetry considerations neatly explain why this is so. For ordinary multiplets, the eigenvalue of H_0 is different from zero; the perturbation H_{sb} only generates a correction, whose magnitude depends on the level in question, because H_{sb} breaks $SU(3)$. In the case of the Goldstone bosons, however, the entire mass is due to the perturbation — the pattern of levels directly reveals the asymmetries of the operator H_{sb} . As discussed above, M_{π^+} is proportional to $\sqrt{m_u + m_d}$. The same analysis applies to the currents $\bar{s}\gamma^\mu\gamma_5 u$ and $\bar{s}\gamma^\mu\gamma_5 d$, which generate transitions from the vacuum to the states $|K^+\rangle$ and $|K^0\rangle$. Since the corresponding divergences are proportional to $(m_u + m_s)$ and $(m_d + m_s)$, one now obtains $M_{K^+} \propto \sqrt{m_u + m_s}$ and $M_{K^0} \propto \sqrt{m_d + m_s}$. The mass ratios of the Goldstone bosons strongly deviate from unity, because m_s happens to be large compared to m_u and m_d .

The level shifts generated by the symmetry breaking may be analyzed by treating the mass term in eq.(10) as a perturbation. To first order in the perturbation, the result obeys the Gell-Mann-Okubo formula. The calculation also applies to the pseudoscalar octet, where the unperturbed levels sit at $M=0$, provided the shifts in the *square* of the mass are considered. Indeed, M_π^2 , M_K^2 and M_η^2 obey the formula remarkably well, confirming that the mass pattern of the pseudoscalar octet is perfectly consistent with the claim that SU(3) is a decent approximate symmetry of the strong interaction.¹

The first order mass formulae for the pseudoscalar octet may also be used to estimate the relative size of the three quark masses [4]. The most remarkable feature of the resulting pattern is that the quark masses strongly break isospin symmetry: m_u and m_d are quite different [5]. This may be verified as follows. Consider the mass difference between K^0 and K^+ . If m_u and m_d were the same, the splitting would exclusively be due to the electromagnetic interaction. Since the main contribution from this interaction is the self energy of the electric field surrounding the K^+ , this particle would have to be heavier than the K^0 . The observed splitting, $M_{K^0} - M_{K^+} = 4$ MeV is of opposite sign. Hence the difference between m_d and m_u must make a significant contribution, opposite to the electromagnetic one, $m_d > m_u$ (the same conclusion also follows from the mass difference between neutron and proton). In first order perturbation theory, the mass ratio $(M_{K^0}^2 - M_{K^+}^2)/M_{\pi^+}^2$ is given by the relative size of isospin breaking in the quark masses, $r = (m_d - m_u)/(m_u + m_d)$. Using the observed meson masses, this gives $r \simeq 0.20$. If the electromagnetic self energy is taken into account, the result becomes even larger, because the two contributions are of opposite sign: $r \simeq 0.29$ [4].

The reason why, nevertheless, isospin is a nearly perfect symmetry of the strong interaction is essentially the same as for the case of SU(3) breaking, discussed above: The relative magnitude of isospin breaking in the quark masses does not represent an adequate estimate for the magnitude of the isospin breaking effects occurring in the bound states. What counts, instead, is the magnitude of the isospin breaking part of the Hamiltonian, \bar{H}_{sb} , compared to the isospin symmetric piece, \bar{H}_0 (see eq.(1)). This is particularly

¹The experimental values of the decay constants F_π, F_K , which represent the bound state wave functions at the origin, also confirm the picture: The asymmetry seen there, $F_K/F_\pi = 1.22$ is quite typical of the SU(3) breaking effects observed in other multiplets.

evident in the case of the nucleon, where the splitting is of the order of 1 MeV, while the isospin invariant part is responsible for the mean mass and is of order 1 GeV. In algebraic terms, the matrix elements of \bar{H}_{sb} are of order $m_d - m_u$, while those of \bar{H}_0 are determined by the scale Λ_{QCD} , so that the magnitude of isospin breaking is determined by the ratio $(m_d - m_u)/\Lambda_{\text{QCD}}$, rather than $(m_d - m_u)/(m_u + m_d)$.

For the kaons, isospin breaking is enhanced, because these particles get their mass from m_s , not from the scale of QCD: the ratio $(M_{K^0} - M_{K^+})/(M_{K^0} + M_{K^+})$ is of order $(m_d - m_u)/m_s$. One might expect that the most important isospin breaking effects occur in the pion multiplet, where the matrix elements of \bar{H}_0 are suppressed even more strongly. It so happens, however, that the strong breaking of SU(3) symmetry seen in the pseudoscalar octet does not repeat itself here, because the matrix elements of the perturbation, $\langle \pi | \bar{H}_{\text{sb}} | \pi \rangle$ are suppressed, too: The mass splitting $M_{\pi^+} - M_{\pi^0}$ is of second order in the perturbation, proportional to $(m_d - m_u)^2$. Numerically, the effect is tiny, of order 0.2 MeV; the observed mass difference is due almost entirely to the electromagnetic interaction. The mathematical origin of this qualitative difference between the two cases is that, in contrast to SU(3), the group SU(2) does not have a d -symbol. For this reason, the pion mass is shielded from isospin breaking, so that the range of the forces generated by pion exchange is nearly charge independent.

7 Effective field theory

At low energies, the behaviour of scattering amplitudes or current matrix elements can be described in terms of a *Taylor series expansion* in powers of the momenta. The electromagnetic form factor of the pion, e.g., may be expanded in powers of the momentum transfer t . In this case, the first two Taylor coefficients are related to the total charge of the particle and to the mean square radius of the charge distribution, respectively,

$$f_{\pi^+}(t) = 1 + \frac{1}{6}\langle r^2 \rangle_{\pi^+} t + O(t^2) . \quad (11)$$

Scattering lengths and effective ranges are analogous low energy constants occurring in the Taylor series expansion of scattering amplitudes.

The occurrence of light particles gives rise to singularities in the low energy domain, which limit the range of validity of the Taylor series representation. The form factor $f_{\pi^+}(t)$, e.g., contains a branch cut at $t = 4M_\pi^2$, such

that the formula (11) provides an adequate representation only for $|t| \ll 4M_\pi^2$. The problem becomes even more acute if m_u and m_d are set equal to zero. The pion mass then disappears, the branch cut sits at $t = 0$ and the Taylor series does not work at all. I first discuss the method used in the low energy analysis for this extreme case, returning to the physical situation with $m_u, m_d \neq 0$ below.

The reason why the spectrum of QCD with two massless quarks contains three massless bound states is understood: they are the Goldstone bosons of a hidden symmetry. The symmetry, which gives birth to these, at the same time also determines their low energy properties. This makes it possible to explicitly work out the poles and branch cuts generated by the exchange of Goldstone bosons. The remaining singularities are located comparatively far from the origin, the nearest one being due to the ρ -meson. The result is a modified Taylor series expansion in powers of the momenta, which works, despite the presence of massless particles. In the case of the $\pi\pi$ scattering amplitude, e.g., the radius of convergence of the modified series is given by $s = M_\rho^2$, where s is the square of the energy in the center of mass system (the first few terms of the series only yield a decent description of the amplitude if s is smaller than the radius of convergence, say $s < \frac{1}{2}M_\rho^2 \rightarrow \sqrt{s} < 540$ MeV).

As pointed out by Weinberg [6], the modified expansion may explicitly be constructed by means of an effective field theory, which is referred to as *chiral perturbation theory* and involves the following ingredients:

- (i) The quark and gluon fields of QCD are replaced by a set of pion fields, describing the degrees of freedom of the Goldstone bosons. It is convenient to collect these in a 2×2 matrix $U(x) \in \text{SU}(2)$.
- (ii) The Lagrangian of QCD is replaced by an effective Lagrangian, which only involves the field $U(x)$, and its derivatives

$$\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{eff}}(U, \partial U, \partial^2 U, \dots) .$$

- (iii) The low energy expansion corresponds to an expansion of the effective Lagrangian, ordered according to the number of the derivatives of the field $U(x)$. Lorentz invariance only permits terms with an even number of derivatives,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}}^2 + \mathcal{L}_{\text{eff}}^4 + \mathcal{L}_{\text{eff}}^6 + \dots$$

Chiral symmetry very strongly constrains the form of the terms occurring in the series. In particular, it excludes momentum independent interaction

vertices: Goldstone bosons can only interact if they carry momentum. This property is essential for the consistency of the low energy analysis, which treats the momenta as expansion parameters. The leading contribution involves two derivatives,

$$\mathcal{L}_{\text{eff}}^2 = \frac{1}{4}F_\pi^2 \text{tr}\{\partial_\mu U^+ \partial^\mu U\} , \quad (12)$$

and is fully determined by the pion decay constant. At order p^4 , the symmetry permits two independent terms,²

$$\mathcal{L}_{\text{eff}}^4 = \frac{1}{4}l_1(\text{tr}\{\partial_\mu U^+ \partial^\mu U\})^2 + \frac{1}{4}l_2 \text{tr}\{\partial_\mu U^+ \partial_\nu U\} \text{tr}\{\partial^\mu U^+ \partial^\nu U\} , \quad (13)$$

etc. For most applications, the derivative expansion is needed only to this order.

The most remarkable property of the method is that it does not mutilate the theory under investigation: The effective field theory framework is no more than an efficient machinery, which allows one to work out the modified Taylor series, referred to above. If the effective Lagrangian includes all of the terms permitted by the symmetry, the effective theory is mathematically equivalent to QCD [6, 7]. It exclusively exploits the symmetry properties of QCD and involves an infinite number of effective coupling constants, F_π, l_1, l_2, \dots , which represent the Taylor coefficients of the modified expansion.

In QCD, the symmetry, which controls the low energy properties of the Goldstone bosons, is only an approximate one. The constraints imposed by the hidden, approximate symmetry can still be worked out, at the price of expanding the quantities of physical interest in powers of the symmetry breaking parameters m_u and m_d . The low energy analysis then involves a combined expansion, which treats both, the momenta and the quark masses as small parameters. The effective Lagrangian picks up additional terms, proportional to powers of the quark mass matrix,

$$m = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}$$

It is convenient to count m like two powers of momentum, such that the expansion of the effective Lagrangian still starts at $O(p^2)$ and only contains

²In the framework of the effective theory, the anomalies of QCD manifest themselves through an extra contribution, the Wess-Zumino term, which is also of order p^4 and is proportional to the number of colours.

even terms. The leading contribution picks up a term linear in m ,

$$\mathcal{L}_{\text{eff}}^2 = \frac{1}{4}F_\pi^2 \text{tr}\{\partial_\mu U^+ \partial^\mu U\} + \frac{1}{2}F_\pi^2 B \text{tr}\{m(U + U^\dagger)\} . \quad (14)$$

Likewise, $\mathcal{L}_{\text{eff}}^4$ receives additional contributions, involving two further effective coupling constants, l_3, l_4 , etc.

The expression (14) represents a compact summary of the soft pion theorems established in the 1960's: The leading terms in the low energy expansion of the scattering amplitudes and current matrix elements are given by the tree graphs of this Lagrangian. The coupling constant B , needed to account for the symmetry breaking effects generated by the quark masses at leading order, represents the coefficient of the linear term in the expansion of the pion mass, $M_\pi^2 = (m_u + m_d)B + O(m^2)$. According to section 5, this constant also determines the vacuum-to-pion matrix element of the pseudoscalar density, $G_\pi = F_\pi B + O(m)$. Furthermore, the relation of Gell-Mann, Oakes and Renner, $F_\pi^2 M_\pi^2 = -(m_u + m_d) \langle 0 | \bar{u}u | 0 \rangle + O(m^2)$, which immediately follows from the above expression for the effective Lagrangian, shows that the magnitude of the quark condensate is also related to the value of B .

The effective field theory represents an efficient and systematic framework, which allows one to work out the corrections to the soft pion predictions, those arising from the quark masses as well as those from the terms of higher order in the momenta. The evaluation is based on a perturbative expansion of the quantum fluctuations of the effective field. In addition to the tree graphs relevant for the soft pion results, graphs containing vertices from the higher order contributions $\mathcal{L}_{\text{eff}}^4, \mathcal{L}_{\text{eff}}^6 \dots$ and loop graphs contribute. The leading term of the effective Lagrangian describes a nonrenormalizable theory, the "nonlinear σ -model". The higher order terms in the derivative expansion, however, automatically contain the relevant counter terms. The divergences occurring in the loop graphs merely renormalize the effective coupling constants. The effective theory is a perfectly renormalizable scheme, order by order in the low energy expansion, so that, in principle, the result of the calculation does not depend on who it is who did it.

8 Universality

The properties of the effective theory are governed by the hidden symmetry, which is responsible for the occurrence of Goldstone bosons. In particular,

the form of the effective Lagrangian only depends on the symmetry group G of the Hamiltonian and on the subgroup $H \subset G$, under which the ground state is invariant. The Goldstone bosons live on the difference between the two groups, i.e., on the quotient G/H . The specific dynamical properties of the underlying theory do not play any role. To discuss the consequences of this observation, I again assume that G is an exact symmetry.

In the case of QCD with two massless quarks, $G = \text{SU}(2)_R \times \text{SU}(2)_L$ is the group of chiral isospin rotations, while $H = \text{SU}(2)$ is the ordinary isospin group. The Higgs model is another example of a theory with spontaneously broken symmetry. It plays a crucial role in the Standard Model, where it describes the generation of mass. The model involves a scalar field $\vec{\phi}$ with four components. The Hamiltonian is invariant under rotations of the vector $\vec{\phi}$, which form the group $G = O(4)$. Since the field picks up a vacuum expectation value, the symmetry is spontaneously broken to the subgroup of those rotations, which leave the vector $\langle 0 | \vec{\phi} | 0 \rangle$ alone, $H = O(3)$. It so happens that these groups are the same as those above, relevant for QCD.³ The fact that the symmetries are the same implies that the effective field theories are identical: (i) In either case, there are three Goldstone bosons, described by a matrix field $U(x) \in \text{SU}(2)$. (ii) The form of the effective Lagrangian is precisely the same. In particular, the expression

$$\mathcal{L}_{\text{eff}}^2 = \frac{1}{4} F_\pi^2 \text{tr}\{\partial_\mu U^\dagger \partial^\mu U\}$$

is valid in either case. At the level of the effective theory, the only difference between these two physically quite distinct models is that the numerical values of the effective coupling constants are different. In the case of QCD, the one occurring at leading order of the derivative expansion is the pion decay constant, $F_\pi \simeq 93$ MeV, while in the Higgs model, this coupling constant is larger by more than three orders of magnitude, $F_\pi \simeq 250$ GeV. At next-to-leading order, the effective coupling constants are also different; in particular, in QCD, the anomaly coefficient is equal to N_c , while in the Higgs model, it vanishes.

³The structure of the effective Lagrangian rigorously follows from the Ward identities for the Green functions of the currents, which also reveal the occurrence of anomalies [7]. The form of the Ward identities is controlled by the structure of G and H in the infinitesimal neighbourhood of the neutral element. In this sense, the symmetry groups of the two models are the same: $O(4)$ and $O(3)$ are *locally* isomorphic to $\text{SU}(2) \times \text{SU}(2)$ and $\text{SU}(2)$, respectively.

As an illustration, I compare the condensates of the two theories, which play a role analogous to the spontaneous magnetization $\langle \vec{M} \rangle$ of a ferromagnet (or the staggered magnetization of an antiferromagnet). At low temperatures, the magnetization singles out a direction — the ground state spontaneously breaks the symmetry of the Hamiltonian with respect to rotations. As the system is heated, the spontaneous magnetization decreases, because the thermal disorder acts against the alignment of the spins. If the temperature is high enough, disorder wins, the spontaneous magnetization disappears and rotational symmetry is restored. The temperature at which this happens is the Curie temperature. Quantities, which allow one to distinguish the ordered from the disordered phase are called *order parameters*. The magnetization is the prototype of such a parameter.

In QCD, the most important order parameter (the one of lowest dimension) is the quark condensate. At nonzero temperatures, the condensate is given by the thermal expectation value

$$\langle \bar{u}u \rangle_T = \frac{\text{Tr}\{\bar{u}u \exp(-H/kT)\}}{\text{Tr}\{\exp(-H/kT)\}} .$$

The condensate melts if the temperature is increased. At a critical temperature, somewhere in the range $140 \text{ MeV} < T_c < 180 \text{ MeV}$, the quark condensate disappears and chiral symmetry is restored. The same qualitative behaviour also occurs in the Higgs model, where the expectation value $\langle \vec{\phi} \rangle_T$ of the scalar field represents the most prominent order parameter.

At low temperatures, the thermal trace is dominated by states of low energy. Massless particles generate contributions which are proportional to powers of the temperature, while massive ones like the ρ -meson are suppressed by the corresponding Boltzmann factor, $\exp(-M_\rho/kT)$. In the case of a spontaneously broken symmetry, the massless particles are the Goldstone bosons and their contributions may be worked out by means of effective field theory. For the quark condensate, the calculation has been done [8], up to and including terms of order T^6 :

$$\langle \bar{u}u \rangle_T = \langle 0 | \bar{u}u | 0 \rangle \left\{ 1 - \frac{T^2}{8F_\pi^2} - \frac{T^4}{384F_\pi^4} - \frac{T^6}{288F_\pi^6} \ln(T_1/T) + O(T^8) \right\} .$$

The formula is exact — for massless quarks, the temperature scale relevant at low T is the pion decay constant. The additional logarithmic scale T_1

occurring at order T^6 is determined by the effective coupling constants l_1, l_2 , which enter the expression (13) for the effective Lagrangian of order p^4 . Since these are known from the phenomenology of $\pi\pi$ scattering, the value of T_1 is also known: $T_1 = 470 \pm 110$ MeV.

Now comes the point I wish to make. The effective Lagrangians relevant for QCD and for the Higgs model are the same. Since the operators of which we are considering the expectation values also transform in the same manner, their low temperature expansions are identical. The above formula thus holds, without any change whatsoever, also for the Higgs condensate,

$$\langle \vec{\phi} \rangle_T = \langle 0 | \vec{\phi} | 0 \rangle \left\{ 1 - \frac{T^2}{8F_\pi^2} - \frac{T^4}{384F_\pi^4} - \frac{T^6}{288F_\pi^6} \ln(T_1/T) + O(T^8) \right\} .$$

In fact, the universal term of order T^2 was discovered in the framework of this model, in connection with work on the electroweak phase transition [9].

The effective Lagrangian of a Heisenberg antiferromagnet is also of the same structure,⁴ so that the above formula even holds for the staggered magnetization, except for one modification: the Clebsch-Gordan coefficients, which accompany the various powers of T are different, because the symmetry groups differ: The Hamiltonian now is invariant under ordinary rotations, $G = O(3)$, while the ground state spontaneously breaks the symmetry to the subgroup $H = O(2)$ of the rotations around the direction singled out by the magnetization.

These examples illustrate the physical nature of effective theories: At long wavelength, the microscopic structure does not play any role. The behaviour only depends on those degrees of freedom, which require little excitation energy. The hidden symmetry, which is responsible for the absence of an energy gap and for the occurrence of Goldstone bosons, at the same time also determines their low energy properties. For this reason, the form of the effective Lagrangian is controlled by the symmetries of the system and

⁴Since the ground state of a magnet fails to be Lorentz invariant, the derivative expansion of the effective Lagrangian contains additional contributions. For a cubic lattice, however, the leading term is of the same form as in relativistically invariant theories, except that the velocity of light is to be replaced by the velocity of propagation for magnons of long wavelength. The low energy properties of a ferromagnet, on the other hand, are quite different. The corresponding effective Lagrangian is dominated by a topological term, related to the fact that the generators of the symmetry acquire nonzero expectation values in the ground state [10].

is, therefore, universal. The microscopic structure of the underlying theory exclusively manifests itself in the numerical values of the effective coupling constants. The temperature expansion also clearly exhibits the limitations of the method. The truncated series can be trusted only at low temperatures, where the first term represents the dominant contribution. According to the above formula, the quark condensate drops to about half of the vacuum expectation value when the temperature reaches 160 MeV — the formula does not make much sense beyond this point. In particular, the behaviour of the quark condensate in the vicinity of the chiral phase transition is beyond the reach of the effective theory discussed here.

9 Experimental aspects

The DAFNE Handbook [11] provides an excellent overview over many of the processes, where new data will contribute to make progress in understanding the low energy structure of QCD. I only add a few comments.

One of the issues, about which very little is known experimentally, is the explicit breaking of chiral and isospin symmetry, generated by m_u and m_d . Because the group $SU(2)_R \times SU(2)_L$ represents an almost exact symmetry of the strong interaction, the symmetry breaking part of the Hamiltonian only generates very small effects. An excellent place to check the theoretical ideas about the implications of symmetry breaking is $\pi\pi$ scattering. As shown by Weinberg [12], nearly 30 years ago, chiral symmetry leads to parameter free soft pion predictions for the corresponding S -wave scattering lengths a_0, a_2 . There is a beautiful proposal [13] to accurately measure the combination $a_0 - a_2$, by producing $\pi^+\pi^-$ atoms and measuring the rate of their decay into $\pi^0\pi^0$. The corrections to the soft pion results have been worked out [14], so that a very accurate prediction is available for test. The S -wave scattering lengths are closely related to the σ -term matrix element $\sigma_{\pi\pi} = \langle \pi | m_u \bar{u}u + m_d \bar{d}d | \pi \rangle$ and are also proportional to $m_u + m_d$. The quantity $a_0 - a_2$ thus represents a direct measure of the asymmetries produced by the quark masses. The experiment, in particular, would provide a sensitive test of the standard hypothesis, according to which the expansion of the pion mass in powers of the quark masses,

$$M_\pi^2 = M^2 \left\{ 1 - \frac{M^2}{32\pi^2 F_\pi^2} \bar{l}_3 + O(M^4) \right\} , \quad M^2 \equiv (m_u + m_d)B ,$$

is dominated by the first term. In the standard picture, the contribution of order $(m_u + m_d)^2$, which is proportional to the effective coupling constant \bar{l}_3 , amounts to a small correction of order 2%; the corresponding contribution to $a_0 - a_2$ is three times smaller. As pointed out by Knecht et al. [15], the arguments which underly this estimate are theoretical: There is no direct experimental evidence, which would rule out an entirely different picture. A number like $\bar{l}_3 = -100$, e.g., would increase the result for $a_0 - a_2$ by about 25% and bring it into agreement with the central value of the currently available data. Conversely, if this value should be confirmed within narrow error bars, one would have to conclude that the "correction" in the expansion of M_π^2 is almost as large as the leading term. Needless to say that this would give rise to a major earthquake in the current understanding of QCD. The quark mass pattern discussed above is based on the standard picture, where it is assumed that the Gell-Mann-Oakes-Renner relation is not ruined by higher order terms. This is the only way I know of to understand the success of the Gell-Mann-Okubo formula for the pseudoscalar octet — if the symmetry breaking observed in $\pi\pi$ scattering should disagree with the theoretical predictions, the standard picture would require thorough revision, even at the qualitative level. Only few of us expect this to be the outcome of the investigation, but the earmark of an important experiment is the product of the likelihood for a discovery with the physical significance thereof, the likelihood as such may be quite small.

The analogous issue in pion-nucleon scattering is a dinosaur. It is notoriously difficult to accurately measure $\sigma_{\pi N}$. At the present time, the experimental uncertainties in this quantity amount to about 20%, comparable to those in the $\pi\pi$ S -wave scattering lengths. There are beautiful new data on the related πN scattering lengths, based on bound states of $\pi^- p$ and $\pi^- d$ [16], analogous to the $\pi^+ \pi^-$ atoms of the proposal mentioned above. These data attain a precision, where even isospin breaking effects due to $m_d - m_u$ can be measured, provided the theoretical results [17], used to express the pion-deuteron scattering lengths in terms of those of proton and neutron, can be trusted at the accuracy needed here. The new data should give ample incentive for a careful reanalysis of the three-body problem, which arises if a pion of zero momentum encounters a deuteron. Evidently, the experimental discrepancies in low energy πN scattering should be resolved. For a measurement of small quantities like $\sigma_{\pi N}$, the dominating contribution from the Born term, i.e., the value of the coupling constant $g_{\pi N}$, needs to be known to

very high precision.

On the theoretical side, considerable progress in the chiral perturbation theory of the πN interaction is being made. The predictions are weaker here, because, in the hidden symmetry game, the nucleons are only spectators, not actors like the Goldstone bosons. Accordingly, the number of effective coupling constants, which need to be taken from phenomenology, is larger. In the case of the σ -term, e.g., the symmetry implies that the matrix element $\langle \pi | \bar{s}s | \pi \rangle$ vanishes if m_u, m_d are sent to zero, while this is not the case for the corresponding nucleon matrix element. Also, the $\pi\pi$ scattering matrix elements are shielded from the perturbations generated by $m_d - m_u$, but the πN scattering matrix elements are not — *small* quantities like the σ -term or the isospin even S -wave scattering length may pick up *comparatively* large charge asymmetries [4]. The fact that the excitation energy of the Δ is relatively small does not really present a problem; unless one attempts to use the effective theory in the vicinity of the resonance or beyond, the corresponding singularity may be expanded in the standard fashion, absorbing the Taylor coefficients in the relevant low energy constants. The expansion of the πN scattering amplitude in powers of the momenta, however, contains odd as well as even powers — one needs to carry the expansion beyond the first two terms to achieve the same precision as the one available for $\pi\pi$ scattering [18]. Work on this problem is of interest, in particular, in connection with the ongoing experiments on pion photo- and electroproduction, whose significance as probes of the low energy structure is becoming increasingly evident and which were discussed in detail at this workshop.

Another topic, where the experimental situation needs to be clarified, is η decay. It is important to resolve the discrepancy between the older data, based on the Primakoff effect and the more recent ones, from photon-photon-collisions. The rate of the decay into three pions measures the ratio $(m_d^2 - m_u^2)/m_s^2$ of quark masses [19]. Also, the available information on the Dalitz plot distribution of the $\pi^+ \pi^- \pi^0$ final state and on the ratio $\Gamma_{\eta \rightarrow 3\pi^0}/\Gamma_{\eta \rightarrow \pi^0 \pi^+ \pi^-}$ leaves to be desired. Incidentally, the world average of the partly inconsistent data on these quantities is not in satisfactory agreement with the theoretical predictions.

There are many other items of interest, which are by no means less interesting — processes generated by the Wess-Zumino term, to only name one category — but I stop here, thanking Aron Bernstein, Barry Holstein and their coworkers for a very informative meeting.

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